VKSU Example

If two sinusoidal waves given as, $y_1 = 0.03 \cos(7t - 10x)$ m and $y_2 = 0.03 \cos(5t - 8x)$ m were superimposed. Find the group velocity.

Sol: Given That

$y_1 = 0.03 \cos(7t - 10x)$ m — (1)

$y_2 = 0.03 \cos(5t - 8x)$ m — (2)

Comparing with the wave equn $y = a \cos(\omega t - kx)$ we have,

Amplitude $a = 0.03$ m

Angular frequency, $\omega_1 = 7$; $\omega_2 = 5$

Propagation constant $k_1 = 10$; $k_2 = 8$

Group velocity $V_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$

$= \frac{7-5}{10-8} = 1 \text{ ms}^{-1}$

Ex Find the group velocity when the two waves $y_1 = 10 \sin(2\pi t - 5x)$ and $y_2 = 15 \sin(5\pi t - 5x)$ are superimposed y_1 and y_2 are in m.

Sol Given That

$y_1 = 10 \sin(2\pi t - 5x)$ — (1)

$y_2 = 15 \sin(5\pi t - 5x)$ — (2)

Comparing with the wave equation $y = a \sin(\omega t - kx)$ we have

Angular frequency $\omega_1 = 2\pi$, $\omega_2 = 5\pi$

Propagation Numbers (constant)

$k_1 = +5$; $k_2 = -5$

$\therefore$ Group velocity $V_g = \frac{\Delta\omega}{\Delta K} = \frac{\omega_1 - \omega_2}{K_1 - K_2}$

$= \frac{3\pi - 5\pi}{+5-(-5)} = \frac{-3\pi}{10}$

$= -0.3\pi$ m/s

Ex Let us Consider That There is a Simple harmonic Wave travelling along X-axis is given by $y = 5\sin 2\pi(0.2t - 0.5x)$ Then find the Value of amplitude, frequency, Wavelength, Wave velocity, Particle velocity, velocity amplitude, Particle accedenation and acceleration amplitude (x is in metres t in Seconds)

Sol Given $y = 5\sin 2\pi(0.2t - 0.5x)$ — (1)

on Comparing with above equation

$y = a\sin\frac{2\pi}{\lambda}(vt - x)$, we obtain

$y = a\sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right) = a\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

Now Amplitude $a = 5$ m

Frequency $n = 0.2\ s^{-1}$

So, $\frac{x}{\lambda} = 0.5x$

$\therefore$ Wavelength $\lambda = \frac{1}{0.5} = 2$ m

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On Putting the Value of n and λ, we obtain

$n\lambda = 0.2 \times 3 = 0.4$ m/s

Particle velocity $= \frac{dy}{dt} = 5\cos 2\pi(0.3t - 0.5x) \times 0.2$

$= \cos 2\pi(0.3t - 0.5x)$ ms^{-1}

Particle velocity amplitude $= 1$ ms^{-1}

Particle acceleration $= \frac{d^2y}{dt^2} = -\sin 2\pi(0.3t - 0.5x)0.2$

$= -0.2 \sin 2\pi(0.3t - 0.5x)$ ms^{-1}

Particle acceleration amplitude $= -0.2$ ms^{-1}

Ex- There are two Sinusoidal waves

$Y_1 = 0.03 \cos(7t - 10x)$ m and

$Y_2 = 0.03 \cos(5t - 8x)$ m

were superimposed find the group velocity.

Sol=

On Comparing above equation with the general equ^n of wave equation $y = a\cos(\omega t - kx)$

we have

Amplitude $a = 0.03$ m

Angular frequency $\omega_1 = 7$; $\omega_2 = 5$

Propagation Constant $k_1 = 10$; $k_2 = 8$

$\therefore$ Group velocity $V_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$

$= \frac{7-5}{10-8} = 1$ ms^{-1}

Ex Find out the phase velocity and group velocity. If the velocity of sea wave is $\sqrt{\left(\frac{g\lambda}{2\pi}\right)}$. Also find the group velocity of these waves when the wavelength is 500 meter.

Sol we have

$$V_p = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{g}{k}} \quad \left(\because k = \frac{2\pi}{\lambda}\right)$$

or $$\frac{\omega}{k} = \sqrt{\frac{g}{k}}$$

or $$\omega = \sqrt{gk}$$

$$\frac{d\omega}{dk} = \sqrt{g}\,\frac{1}{2}k^{-1/2} = \frac{1}{2}\sqrt{\frac{g}{k}}$$

Group velocity,

$$\frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}\sqrt{\frac{g\lambda}{2\pi}}$$

or $$V_g = \frac{1}{2}V_p ;$$

$$V_p = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{9.8 \times 500}{2 \times 3.14}}$$

$$= 27.93 \text{ m/sec.}$$

$$V_g = \frac{V_p}{2} = \frac{27.93}{2} = 13.96 \text{ m/sec}$$